Quadratic Equations

Assertion & Reason Type Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 1. Assertion (A): $(3x-2)^2 - 9x^2 + 10 = 0$ is a quadratic equation.

Reason (R): x = 0, 4 are the roots of the equation

$3x^2 - 12x = 0$.

Answer : (d) **Assertion (A):** Given equation is

$(3x-2)^2 - 9x^2 + 10 = 0$

$9x^2 + 4 - 12x - 9x^2 + 10 = 0$

$14 - 12x = 0$, which is not a quadratic equation.

Thus, Assertion (A) is false.

Reason (R): Given quadratic equation is $3x^2 - 12x = 0$.

=> $3x(x-4) = 0$

=> $X = 0, 4$

Thus, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

Q 2. Assertion (A): If $5+\sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5-\sqrt{7}$.

Reason (R): Surd roots of a quadratic equation with rational coefficients occur in conjugate pairs.

Answer : (a) The quadratic formula, which was derived by completing the square, tells us:

$$x_1 = \frac{-b}{2a} + \frac{\sqrt{D}}{2a} \text{ and } x_2 = \frac{-b}{2a} - \frac{\sqrt{D}}{2a}$$

where $(D) = b^2 - 4ac$

Here, x, and x^2 are real conjugates of one another if

(D) is positive but not a perfect square.

Given, one of the roots of quadratic equation having

rational coefficients is 5+√7.

So, the second root will be 5-√7.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q 3. Assertion (A): One of the root of the equation

$2x^2 + 5x - 3 = 0$ **is** $\frac{1}{2}$.

Reason (R): Roots of the quadratic equation $ax^2 + bx + c = 0$ can be determined by using the

formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Answer : (a) **Assertion (A):** Given equation is $2x^2+5x-3=0$.

On comparing with $ax^2 + bx + c = 0$, we get

a=2, b=5 and c=-3

By using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(5)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$= \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm \sqrt{49}}{4}$$

$$= \frac{-5 \pm 7}{4} = \frac{-5+7}{4} \text{ or } \frac{-5-7}{4}$$

$$= \frac{2}{4} \text{ or } -\frac{12}{4} = \frac{1}{2} \text{ or } -3$$

Thus, one of the root of the given equation is $\frac{1}{2}$.

So, Assertion (A) is true.

Reason (R): It is also true that roots are determined

by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q 4.

Assertion (A): The value of $k = -\frac{1}{4}$, if one root of the quadratic equation $5x^2 - x + 3k = 0$ is $\frac{1}{2}$.

Reason (R): The quadratic equation $ax^2 + bx + c = 0$, a=0 has atmost two roots.

Answer : (b) **Assertion (A):** Given quadratic equation is

$$5x^2 - x + 3k = 0$$

Since, $\frac{1}{2}$ is a root of the equation $5x^2 - x + 3k = 0$.

Therefore, put $x = \frac{1}{2}$, we get

$$5\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 3k = 0$$

$$\Rightarrow \quad \frac{5}{4} - \frac{1}{2} + 3k = 0$$

$$\Rightarrow \quad 3k = \frac{1}{2} - \frac{5}{4} \quad \Rightarrow \quad 3k = \frac{2-5}{4}$$

$$\Rightarrow \quad k = \frac{-3}{3 \times 4} \quad \Rightarrow \quad k = -\frac{1}{4}$$

Thus, Assertion (A) is true.

Reason (R): It is also true to say that any quadratic equation has atmost two roots.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Q 5. Assertion (A): The roots of the quadratic equation $x^2+5x+7=0$ are imaginary.

Reason (R): In quadratic equation $ax^2 + bx + c = 0$, if $D-b^2-4ac<0$, then roots are said to be imaginary.

Answer : (a) **Assertion (A):** Given quadratic equation is

$x^2 +5x+7=0$

On comparing with $ax^2 + bx+c=0$, we get

$a=1$, $b = 5$ and $c=7$

Now, discriminant $D=b^2-4ac$

$= (5)^2 - 4x1x7$

$=25-28=-3 <0$

So, roots of given equation are imaginary.

Thus, Assertion (A) is true.

Reason (R): It is also true that, the condition of imaginary root is $D < 0$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q 6. Assertion (A): The equation $9x^2 + 3kx + 4 = 0$ has equal roots for $k = \pm 4$.

Reason (R): If discriminant 'D' of a quadratic equation equals to zero, then the roots of quadratic equation are real and equal.

Answer : (a) **Assertion (A):** Given equation is

$9x^2+3kx+4=0$

On comparing with $ax^2 + bx + c = 0$, we get

$a=9$, $b=3k$ and $c = 4$.

The condition for equal roots is $D= 0$.

$b^2-4ac=0$

$(3k)^2-4x9x4=0$

$9k^2-9x16-0$

$k^2 = 16$

Thus, Assertion (A) is true.

$k=+4$

Reason (R): It is also true and it is the correct explanation of Assertion (A).

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q.7. Assertion (A) : $4x^2 - 12x + 9 = 0$ has repeated roots.

Reason (R) : The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant D > 0.

Answer : (c)

$$\begin{aligned} 4x^2 - 12x + 9 &= 0 \\ D &= b^2 - 4ac \\ &= (-12)^2 - 4(4)(9) \\ &= 144 - 144 = 0 \end{aligned}$$

Roots are repeated.

Q.8. Assertion (A) : The equation $x^2 + 3x + 1 = (x-2)^2$ is a quadratic equation.

Reason (R) : Any equation of the form $ax^2 + bx + c = 0$ where a ≠ 0 is called a quadratic equation.

Answer : (d)

We have, $x^2 + 3x + 1 = (x-2)^2 = x^2 - 4x + 4$

$\Rightarrow \quad x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow \quad 7x - 3 = 0,$

it is not of the form $ax^2 + 6x + c = 0$

So, A is incorrect but R is correct.

Q.9. Assertion (A) : $(2x-1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.

Reason (R) : x = 0, 3 are the roots of the equation $2x^2 - 6x = 0$.

Answer : (b)

Assertion and Reason both are true statements. But Reason is not the correct explanation.

Assertion $\quad (2x-1)^2 - 4x^2 + 5 = 0$

$-4x + 6 = 0$

Reason $\quad 2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0$

and $\quad x = 3$

Q.10. Assertion (A) : The values of x are $-\frac{a}{2}$, a for a quadratic equation $2x^2 + ax - a^2 = 0$.

Reason (R) : For quadratic equation $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Answer : (d) Assertion (A) is false but reason (R) is true.

$$2x^2 + ax - a^2 = 0$$

$$x = \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$

$$= \frac{-a + 3a}{4} = \frac{2a}{4}, \frac{-4a}{4}$$

$$x = \frac{a}{2}, -a$$

So, A is incorrect but R is correct.

Q.11. Assertion (A) : The equation $8x^2 + 3kx + 2 = 0$ has equal roots then the value of k is $\pm \frac{8}{3}$.

Reason (R) : The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$

Answer : (a)

$$8x^2 + 3kx + 2 = 0$$

Discriminant, $D = b^2 - 4ac$

$$D = (3k)^2 - 4 \times 8 \times 2 = 9k^2 - 64$$

For equal roots, $D = 0$

$$9k^2 - 64 = 0$$

$$9k^2 = 64$$

$$k^2 = \frac{64}{9}$$

$$k = \pm \frac{8}{3}$$

So, A and R both are correct and R explains A.

Q.12. Assertion (A) : The value of k = 2, if one root of the quadratic equation

$6x^2 - x - k = 0$ is $\frac{2}{3}$

Reason (R) : The quadratic equation $ax^2 - bx + c = 0, a \neq 0$ has two roots.

Answer : (b)

$$\text{As one root is } \frac{2}{3} \quad x = \frac{2}{3}$$

$$6 \times \left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$

$$6 \times \frac{4}{9} - \frac{2}{3} = k$$

$$k = \frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$$

$$k = 2$$

So, both A and R are correct but R does not explain A.

Q.13. Assertion (A) : The roots of the quadratic equation $x^2 - 2x + 2 = 0$ are imaginary.

Reason (R) : If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

Answer : (a) $x^2 + 2x + 2 = 0$

$$\text{Discriminant,} \quad D = b^2 - 4ac$$

$$= (2)^2 - 4 \times 1 \times 2$$

$$= 4 - 8 = - < 04$$

Roots are imaginary. So, both A and R are correct and R explains A.

Q.14. Assertion (A) : If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$

Reason (R) : If a, b, c are odd integer then the roots of the equation $4abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

Answer : (b)

Assertion : Given equation

$$x^2 - bx + c = 0$$

Let α, β be two roots such that

$$|\alpha - \beta| = 1$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 1$$

$$b^2 - 4c = 1$$

Reason : Given equation

$$4abc\,x^2 + (b^2 - 4ac)x - b = 0$$

$$D = (b^2 - 4ac)^2 + 16ab^2c$$

$$D = (b^2 - 4ac)^2 > 0$$

Hence roots are real and unequal.

Q.15. Assertion (A) : The equation $9x^2 + 3kx + 4 = 0$ has equal roots for k = ± 4.

Reason (R) : If discriminant 'D' of a quadratic equation is equal to zero then the roots of equation are real and equal.

Answer : (a) Assertion $9x^2 + 3kx + 4 = 0$

$$D = b^2 - 4ac$$

$$= (3k)^2 - 4(9)(4)$$

$$= 9k^2 - 144$$

For equal roots $D = 0$

$$9k^2 = 144$$

$$k = \pm\frac{12}{3}$$

$$\text{k} = \pm 4$$

Q.16. Assertion (A) : A quadratic equation $ax^2 + bx + c = 0$, has two distinct real roots, if $b^2 - 4ac > 0$.

Reason (R) : A quadratic equation can never be solved by using method of completing the squares.

Answer : (c)

Q.17. Assertion (A) : Sum and product of roots of $2x^2 - 3x + 5 = 0$ are $\frac{3}{2}$ and $\frac{5}{2}$ respectively.

Reason (R) : If α and β are the roots of $ax^2 + bx + c = 0$, a ≠ 0 , then sum of roots = $\alpha + \beta = -\frac{b}{a}$ and product of roots = $\alpha\beta = \frac{c}{a}$.

Answer : (a) Assertion and Reason both are correct and Reason is correct explanation.

$$\text{Assertion} \quad 2x^2 - 3x + 5 = 0$$

$$\alpha + \beta = \frac{-b}{a}$$

$$= \frac{-(-3)}{2} = \frac{3}{2}$$

$$\text{and} \quad \alpha\beta = \frac{c}{a} = \frac{5}{2}$$